Year: 13 Subject: Further Maths A level	**Curriculum Intent:** Some pure topics from A Level mathematics and year 12 are studied in greater depth, while some new topics are introduced. Algebraic work with series is extended. Complex numbers are developed and lead to solutions of problems in algebra, geometry and trigonometry. Matrices are used to solve systems of equations and to explore transformations. Vector methods are applied to problems involving lines and planes. Calculus techniques are extended, including the use of hyperbolic functions and polar coordinates, and culminate in the solution of differential equations. In mechanics, basic principles of forces and their moments, work and energy, impulse and momentum and centres of mass are used to model various situations, including: rigid bodies in equilibrium; particles moving under gravity or on a surface; bodies colliding with direct impact.					
	Term 1		**Term 2**			
Topic Titles (in order of delivery)	1. Further Vectors 2. Series and Induction 3. Further Calculus 4. Maclaurin Series	1. Work, Energy, Power 2. Impulse and Momentum 3. Polar Coordinates 4. First Order Differential Equations	1. Further Complex Numbers 2. Hyperbolic Functions	1. Moments 2. Centres of Mass 3. Second Order Differential Equations	1. Review and Revise	1. Review and Revise
Key knowledge / Retrieval topics	**1. Further Vectors:** • Be able to use the vector product in component form to give a vector perpendicular to two given vectors. • Be able to use the alternative form for the vector product. Know the significance of $a \times b = 0$. • Be able to find the distance between two parallel lines and the shortest distance between two skew lines. • Be able to find the distance from a point to	1. **Work, Energy, Power**: • Understand the language relating to work, energy and power. • Be able to calculate the work done by a force which moves along its line of action. • Be able to calculate the work done by a force which moves at an angle to its line of action. • Be able to calculate kinetic energy.	1. **Further Complex Numbers**: • Understand and use de Moivre's theorem. • Be able to apply de Moivre's theorem to finding multiple angle formulae and to summing suitable series. • Understand the definition $e^{i} = \cos\theta + i\sin\theta$ and hence the form $z = re^{i\theta}$. • Know that every non-zero complex	1. **Moments**: • Be able to draw a force diagram for a rigid body. • Understand that a system of forces can have a turning effect on a rigid body. • Know the meaning of the term couple. • Be able to calculate the moments about a fixed axis of forces acting on a body.	1.	1.

	a line in 2 or 3 dimensions. • Be able to find the distance from a point to a plane. **2. Series and Induction:** • Be able to prove mathematical results by contradiction. • Be able to construct and present a proof using mathematical induction. • Know the difference between a sequence and a series. • Know the meaning of the word converge when applied to either a sequence or a series. • Be able to sum a simple series using partial fractions. 3. **Further Calculus**: • Evaluate improper integrals where either the integrand is undefined at a value in the interval of integration or the interval of integration extends to infinity. • Be able to use the method of partial fractions in integration, including where the denominator has a quadratic factor of form $ax^2 + c$ and one linear term. • Understand the definitions of inverse trigonometric functions.	• Be able to calculate gravitational potential energy. • Understand when the principle of conservation of energy may be applied and be able to use it appropriately. • Understand and use the work-energy principle. • Understand and use the concept of the power of a force as the rate at which it does work. 2. **Impulse and Momentum**: • Be able to calculate the impulse of a force as a vector and in component form. • Understand and use the concept of linear momentum and appreciate that it is a vector quantity. • Understand and use the impulse-momentum equation. • Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.	number has n distinct nth roots, and that on an Argand diagram these are the vertices of a regular n-gon. • Know that the distinct *n*th roots of $re^{i\theta}$ are: $$r^{\frac{1}{n}}\left[cos\left(\frac{\theta+2k\pi}{n}\right)+isin\left(\frac{\theta+2k\pi}{n}\right)\right]$$ For k=0,1 ….. n-1. • Be able to explain why the sum of all the nth roots is zero. • Understand the effect of multiplication by a complex number on an Argand diagram. • Be able to represent complex roots of unity on an Argand diagram. • Be able to apply complex numbers to geometrical • problems. **2. Hyperbolic Functions:** • Understand the definitions of hyperbolic functions, know their domains and ranges and be able	• Be able to calculate the moment of a couple. • Understand and be able to apply the conditions for equilibrium of a rigid body. • Be able to identify whether equilibrium will be broken by sliding or toppling. 2. **Centres of Mass**: • Be able to find the centre of mass of a system of particles of given position and mass. • Know how to locate centre of mass by appeal to symmetry. • Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina. • Be able to find the centre of mass of a composite body by considering each constituent part as a particle at its centre of mass. • Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.		

	• Be able to differentiate inverse trigonometric functions. **4. Maclaurin Series:** • Be able to find the Maclaurin series of a function, • including the general term. • Know that a Maclaurin series may converge only for a restricted set of values of x. • Be able to recognise and use the Maclaurin series of standard functions: $e^x, \ln(1+x)$, • $\sin(x)$, $\cos(x)$ • and $(1+x)^n$	• Understand the term direct impact and the assumptions made when modelling direct impact collisions. • Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies. • Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact. • Understand the significance of e = 0. • Be able to apply Newton's Experimental Law in modelling direct impacts. • Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law. • Understand the significance of e = 1. • Understand that when e < 1 kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy. 3. **Polar Coordinates**:	to sketch their graphs. • Understand and use the identity $\cosh(2x) - \sinh(2x) = 1$. • Be able to differentiate and integrate hyperbolic functions. • Understand and be able to use the definitions of the inverse hyperbolic functions and know their domains and ranges. • Be able to derive and use the logarithmic forms of the inverse hyperbolic functions. • Recognise integrals of functions of the form $(a^2 + x^2)^{-\frac{1}{2}}$ and $(a^2 - x^2)^{-\frac{1}{2}}$ and be able to integrate related functions by using trigonometric substitutions. 3. **Applications of Integration**: • Be able to derive formulae for and calculate the volumes of the	3. **Second Order Differential Equations**: • Know the language of kinematics, and the relationships between the various variables. • Know Newton's 2nd law of motion. • Use differential equations in modelling in kinematics and in other contexts. • Be able to solve differential equations of the form $y'' + ay' + b = 0$, using the auxiliary equation. • Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation. • Be able to solve differential equations of the form $y'' + ay' + b = f(x)$, by solving the homogeneous case and adding a particular integral to the complimentary function.		

		• Understand and use polar coordinates (r, θ) and be able to convert from polar to cartesian coordinates and vice-versa. • Be able to sketch curves with simple polar equations where r is given as a function of θ. • Be able to find the area enclosed by a polar curve **4. First Order Differential Equations** • Understand how to introduce and define variables to describe a given situation in mathematical terms. • Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations. • Know the difference between a general solution and a particular solution. Be able to find both general and particular solutions. • Recognise differential equations where the integrating factor method is appropriate. • Be able to find an integrating factor and understand its	solids generated by rotating a plane region about the x-axis or the y-axis. • Understand and evaluate the mean value of a function.	• Be able to find particular integrals in simple cases. Understand the relationship between different cases of the solution and the nature of the roots of the auxiliary equation. • Be able to solve the equation for simple harmonic motion, $\ddot{x} = -\omega^2 x$ and be able to relate the solution to the motion. • Be able to model damped oscillations using 2nd order differential equations. • Be able to interpret the solutions of equations modelling damped oscillations in words and graphically. • Analyse and interpret model situations with one independent variable and two dependent variables which lead to coupled 1st order simultaneous linear differential		

		significance in the solution of an equation. • Be able to solve an equation using an integrating factor and find both general and particular solutions.		equations and find the solution.		
Understanding / Sequence of delivery	1. Building on prior knowledge and making connections between topics.					
Assessment	End of Topic Assessed Homework Exam Style Questions Grade Boundaries based on A Level 2019		End of Topic Assessed Homework and Practice Papers Exam Style Questions Grade Boundaries based on A Level 2019		Practice Papers Grade Boundaries based on A Level 2019	
	POP Test Past Exam Questions Grade Boundaries based on A Level 2019		PPE Past Exam Questions Grade Boundaries based on A Level 2019		A Level Exams	