Further Maths at Denbigh

Year 11 into Year 12 summer work

Congratulations on choosing to study Further Maths at A level. It is a great course but a challenging one that requires you to have excellent mathematical knowledge and to be able to apply your skills in unfamiliar contexts.

Complete the questions in the booklet and check your answers. If you get any wrong, spend time revising those topics and try them again. It is vital that you are confident in these skills to be successful at A level Further Maths.

Number

1) Which of the following are integers?

3 -2.8 0.4 $\frac{3}{4}$ 7.92 -9 202 0

2) Which of the following values are rational and which are irrational?

$4.\dot{7}$ π $\sqrt{8}$ $\frac{1}{5}$ -7 $\sqrt{16}$ 12.452 3.1

3) If $0 < x < 1$, compare the size of $x - \frac{1}{x}$ to $x^2 - x$

Indices, expanding and factorising

4) If $2^{2x+1} \times 4^{x+1} = 8^{x+2}$, find the value of x

5) Factorise the following:

$5y(y-1) + 3(y-1)$ $pq^2 - p^2q$ $16m^2 - 81n^2$

6) Multiply out the brackets and simplify where possible:
$(x-2)(2x+3)(x+7)$ $-(x-3)(x-1)(3x+2)$ $(x+1)(x-1)(x+5)(4x-1)$

7) $(x-3)(2x+1)(Ax+1) \equiv 8x^3 + Bx^2 + Cx - 3$ Work out the value of A, the value of B and the value of C.

Inequalities

8) Solve the following:

$8x + 3 \leq 4x$ $3(4-x) > 3$ $3x^2 + 2 < 14$

$7x^2 - 4 \geq 59$ $x^2 - 4x + 10 \geq 2x + 5$

9). Draw a set of axes, show the region that satisfies the following inequalities:

$y > 3x - 2$ $y < x + 2$ $y + x > -1$

Functions and Proof

10) $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$

Evaluate $f(4)$ Find $fg(x)$ Find $f^{-1}(x)$

11). $f(x) = 3x^3 - 2x^2 + 4$ Express $f(x+2)$ in the form $ax^3 + bx^2 + cx + d$

12) a) Express $x^2 + 6x + 11$ in the form $(x+a)^2 + b$ where a and b are integers
b) Hence, prove that $x^2 + 6x + 11$ is always positive

Drawing graphs and transformations of curves

13) A curve has the equation $y = 2x^2 - 5x + 12$

a) Write the curve in the for y = $a\,(x+m)^2 + n$ and hence find the minimum points of the graph.

b) Does the graph cross the x-axis? If yes, find the coordinates of the point of intersection.

14). On separate axes, sketch the following graphs:

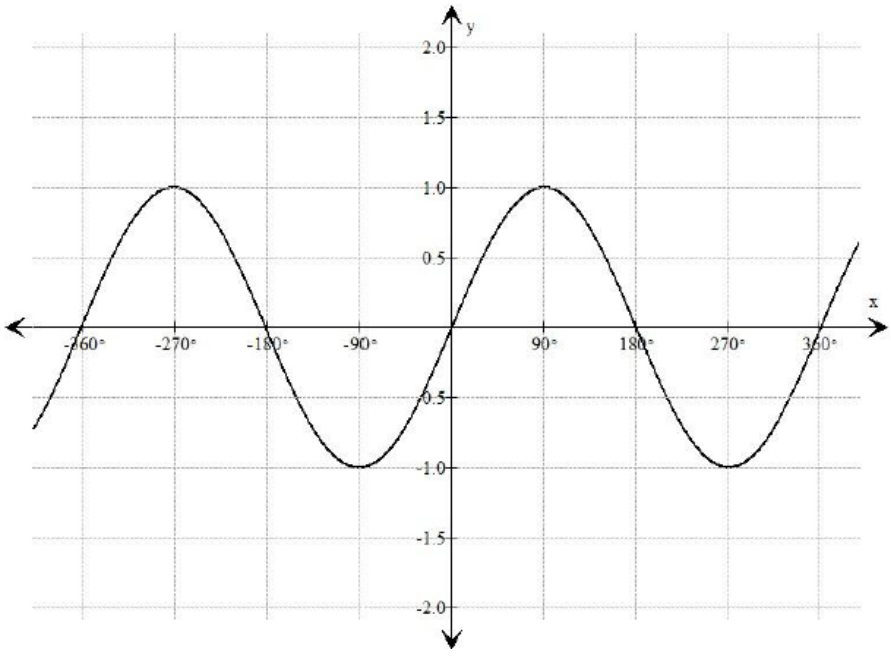
a) $y = -x^3$ $b)$ $y = \frac{-3}{x}$ c) $y = \frac{1}{x} + 1$ d) $y = \frac{2}{x^2}$

15) The graph of $y = \sin(x)$ is plotted below. Sketch the following transformations of $y = \sin(x)$on the same set of axes:

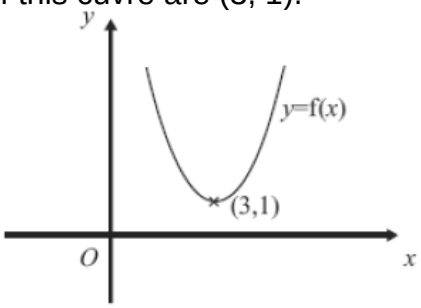
a) $y = 2\sin(x)$

b) $y = \sin(4x)$

c) $y = \sin(x - 90)$



16) The diagram shows part of the curve with equation $y = f(x)$. The coordinates of the minimum point of this cuvre are (3, 1).



Write down the coordinates of the minimum point of the curve with equation:

a) $y = f(x) + 3$

b) $y = f(x - 2)$

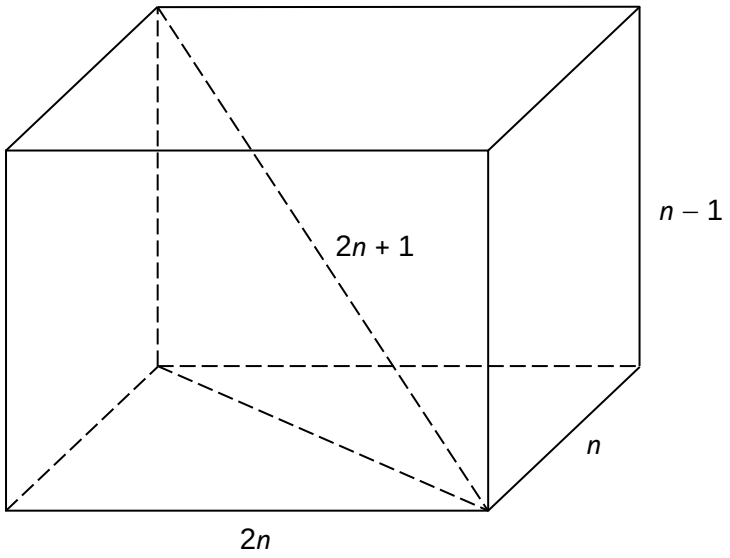
c) $y = f\left(\frac{1}{2}x\right)$

3D Trigonometry and Pythagoras' Theorem

17) A cuboid has dimensions $2n$, n and $n - 1$ cm.

A diagonal has length $2n + 1$ cm.

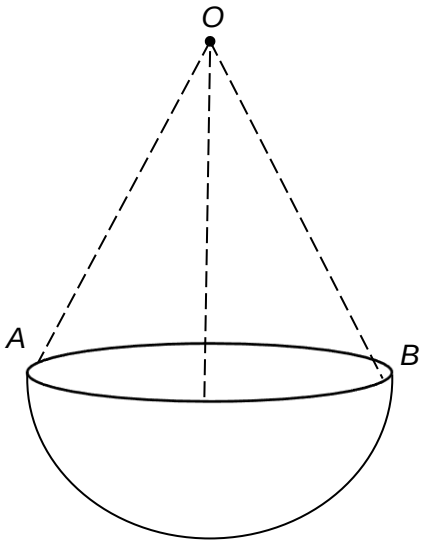
Not drawn accurately



Work out n.

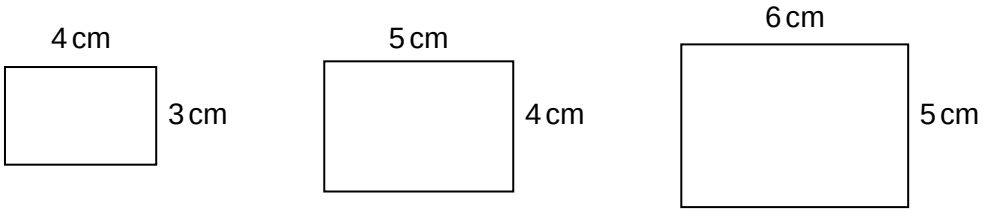
18) A hanging basket is made from a hemisphere and three chains.
The radius of the hemisphere is 10 cm.
Each chain is 30 cm long.
The chains are equally spaced around the rim of the hemisphere.

Work out angle *AOB*.



Sequences

19)



This pattern of rectangles continues.

Show that the sequence of numbers formed by the areas of these rectangles has *n*th term

$$n^2 + 5n + 6$$

20)

A linear sequence starts

$$a + b \qquad a + 3b \qquad a + 5b \qquad a + 7b \qquad \ldots\ldots\ldots\ldots$$

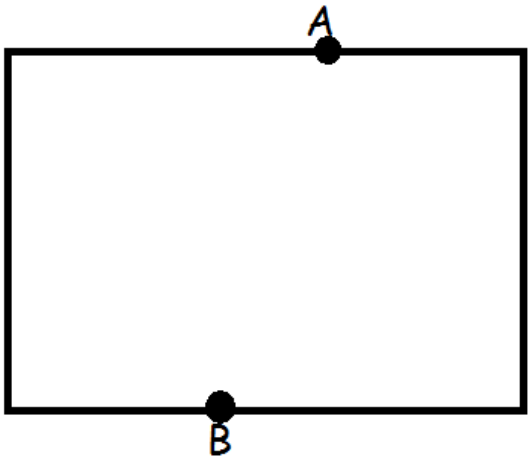
The 5th and 8th terms have values 35 and 59.

(a) Work out *a* and *b*.

(b) Work out the *n*th term of the sequence.

Transformations and Loci

21)



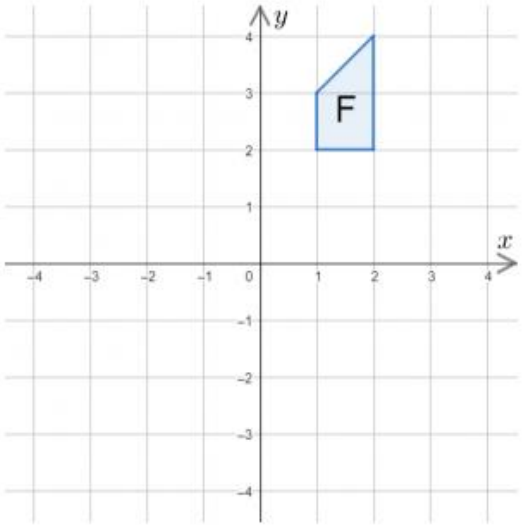
A snail moves so that it is always within the rectangle and is equidistant from points A and B. Use ruler and compasses to show where the snail moves.

22) In this order, perform the following two transformations to shape F.

a) Rotation $180°$ clockwise about (1,2)

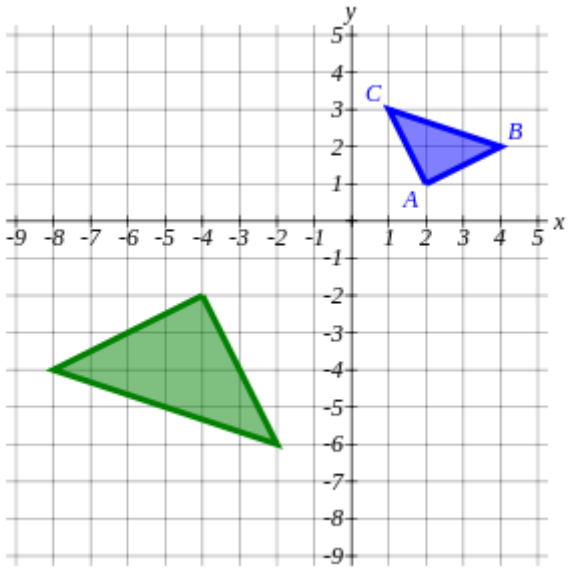
b) Reflection in the line $y = x$

Mark the resulting shape with a G. Extension: Are there any invariant points?



23)

Fully describe the single transformation from the triangle ABC to its image.



Vectors

24)

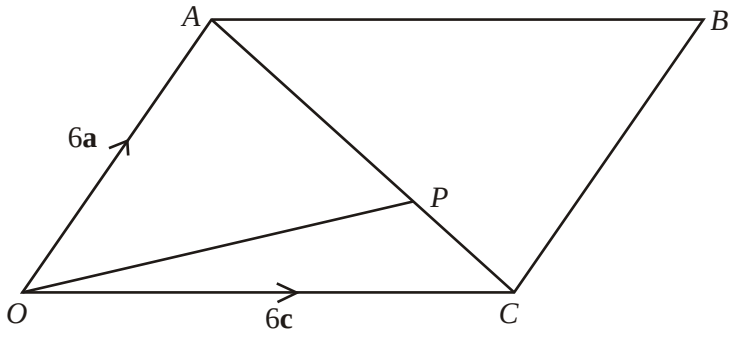


Diagram **NOT** accurately drawn

OABC is a parallelogram.

P is the point on *AC* such that $AP = \frac{2}{3}AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$.
Give your answer in terms of **a** and **c**.

The midpoint of *CB* is *M*.

(b) Prove that *OPM* is a straight line.

Answers

Number

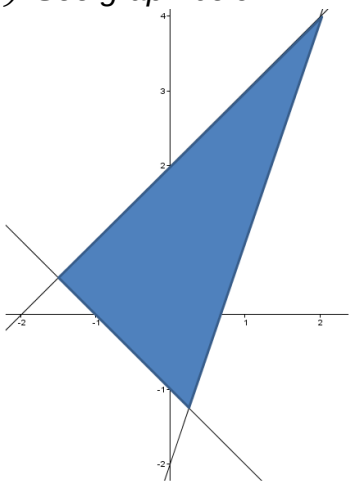
1) 3, -9, 202, 0
2) Rational: 4.7, 1/5, -7 $\sqrt{16}$, 12.451, 3.1
   Irrational: $\sqrt{8}$, π
3) $\left(x-\frac{1}{x}\right)>\left(x^2-x\right)$

Indices, expanding and factorising

4) $x = 3$
5) $(y-1)(5y+3)$ $pq(q-p)$ $(4m+9n)(4m-9n)$
6) $2x^3+13x^2-13x-42$ $-3x^3+10x^2-x-6$ $4x^4+19x^3-9x^2-19x+5$
7) $A=4$ $B=-18$ $C=-17$

Inequalities

8) $x \leq -\frac{3}{4}$ $x<3$ $-2<x<2$ $x \geq 3$ *or* $x \leq -3$ $x \leq 1$ *or* $x \geq 5$
9) See graph below



Functions and Proof

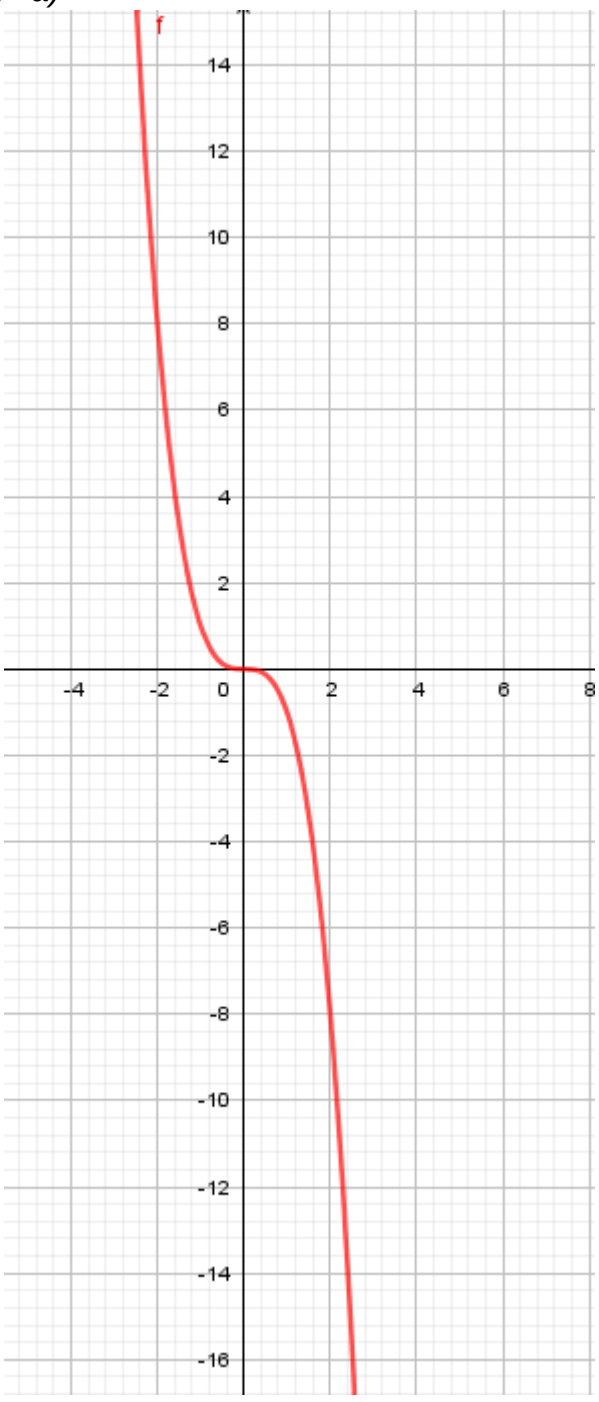
10) $f(4)=3$ $fg(x)=\frac{x+2}{3}$ $f^{-1}(x)=3x-5$
11) $f(x+2)=3x^3+16x^2+28x+20$
12) a =2, b = 3, $(x+3)^2 \geq 0$ *and so adding 2 means always positive.*

Drawing graphs and transformations of curves

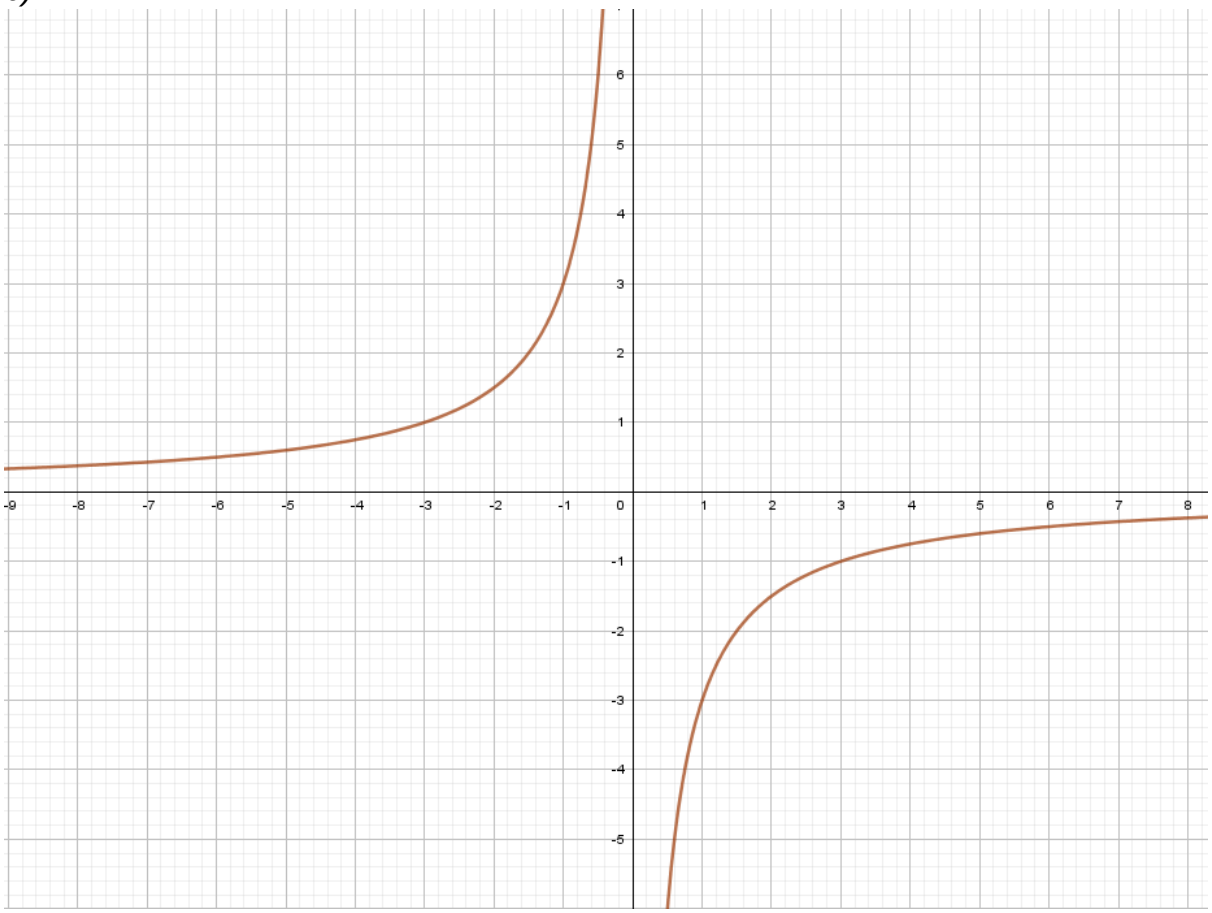
13) a) $y=2(x-\frac{5}{4})^2+\frac{71}{8}$ *so minimum point is* $(\frac{5}{4},\frac{71}{8})$

b) Curve does not intersect x-axis as minimum point is above it and the curve is always positive.

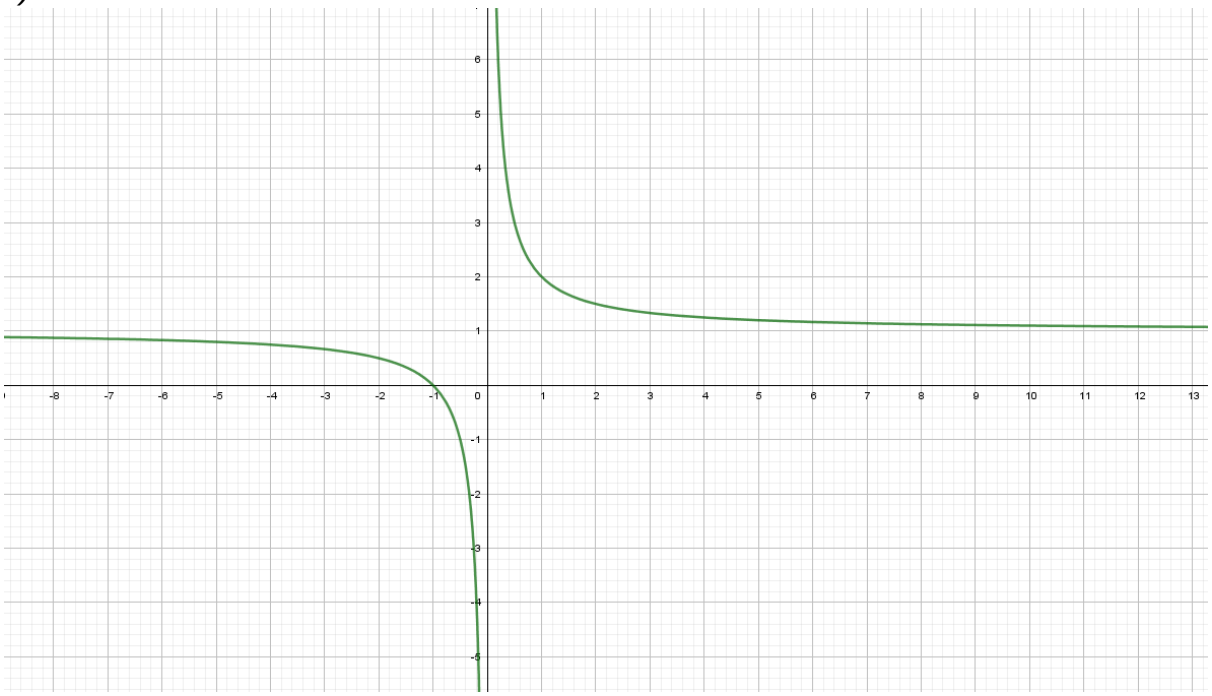
14) a)



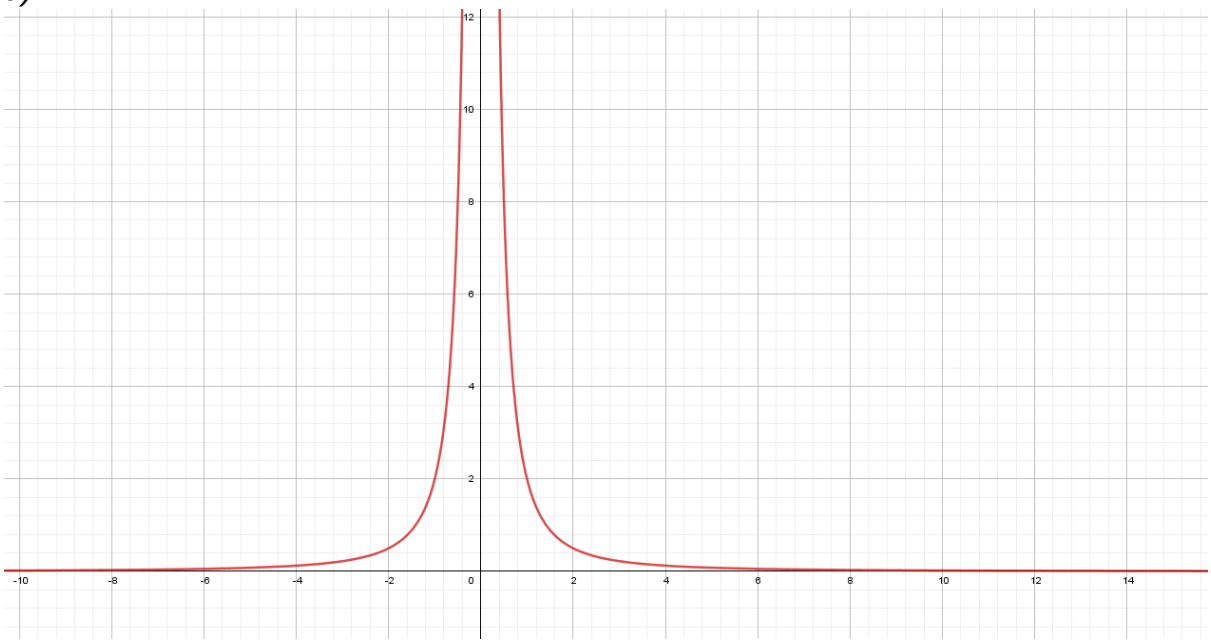
b)


-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8
6 5 4 3 2 1 -1 -2 -3 -4 -5

c)

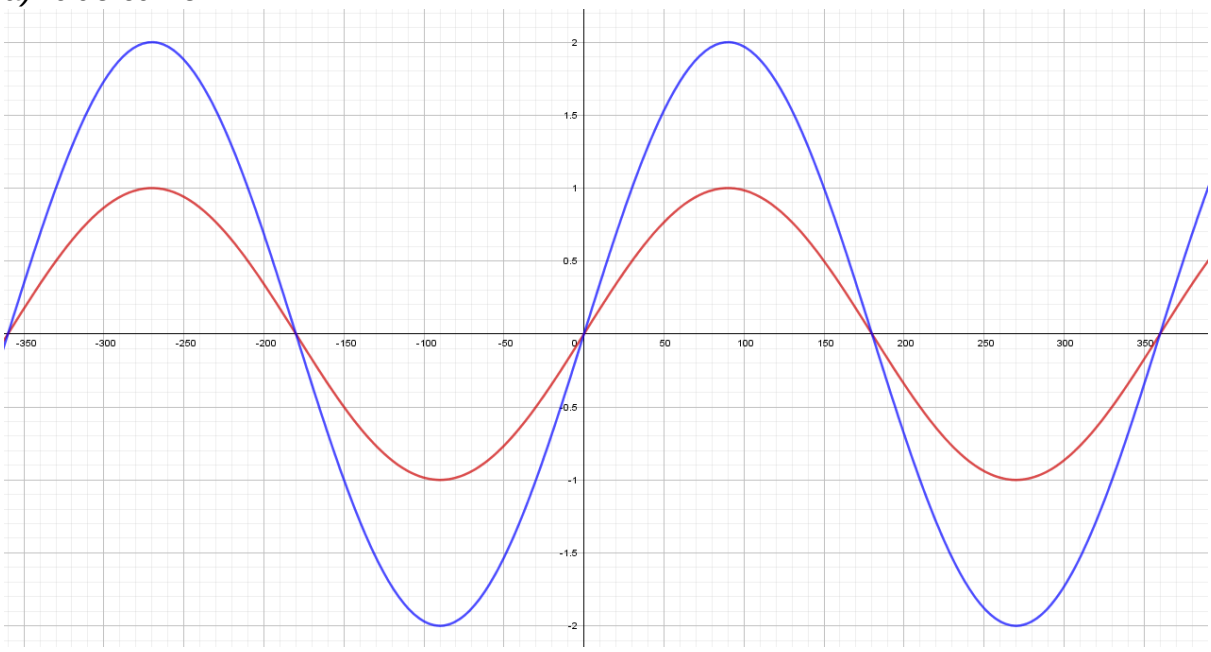

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13
6 5 4 3 2 1 -1 -2 -3 -4 -5

d)

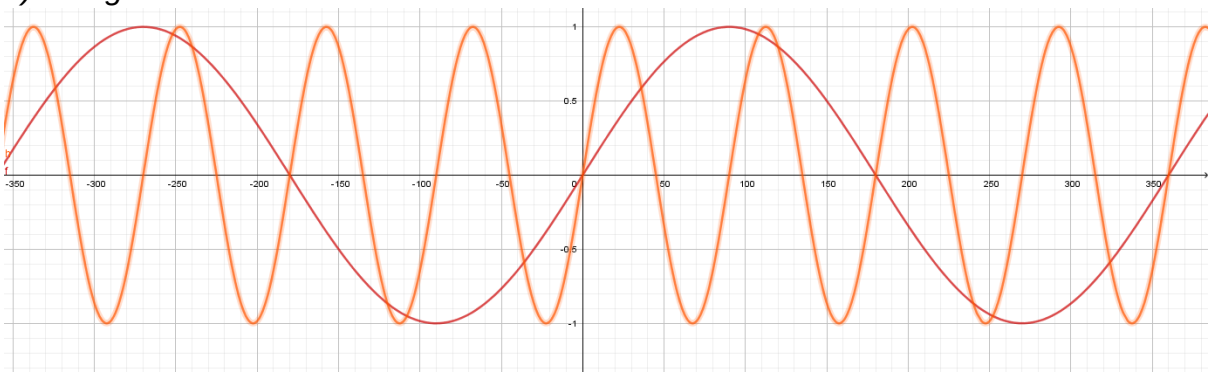


15)

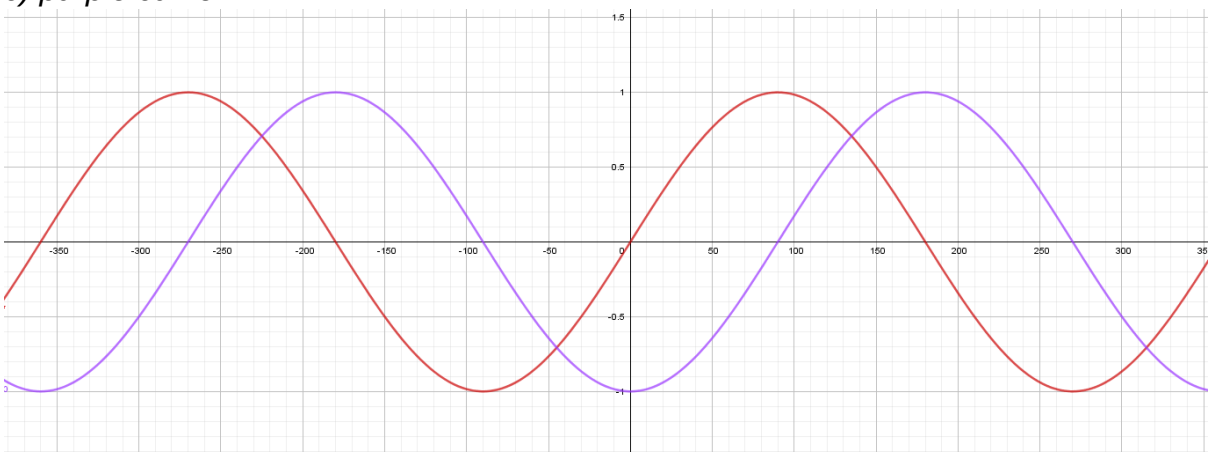
a) blue curve



b) orange curve



c) purple curve



16) a) (3,4) b) (5,1) c) (6,1)

3D Trigonometry and Pythagoras' Theorem

17) Workings in the table below

$(2n)^2 + n^2$

$(2n)^2 + n^2 + (n - 1)^2 = (2n + 1)^2$

$4n^2 + n^2 + n^2 - n - n + 1$

$= 4n^2 + 2n + 2n + 1$

$2n^2 - 6n = 0$

$2n(n - 3) = 0$

$n = 3$

18) Workings in the table below

A triangle formed with A, B and the centre of the hemisphere with 2 sides of 10 cm and an angle of 120°

$(AB^2 =)\ 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120$

$(AB =)$ [17.3, 17.321]

$(\cos AOB =)\ \dfrac{30^2 + 30^2 - \text{their } AB^2}{2 \times 30 \times 30}$

[33.557, 33.6]

Sequences

19) *Workings in the table below*

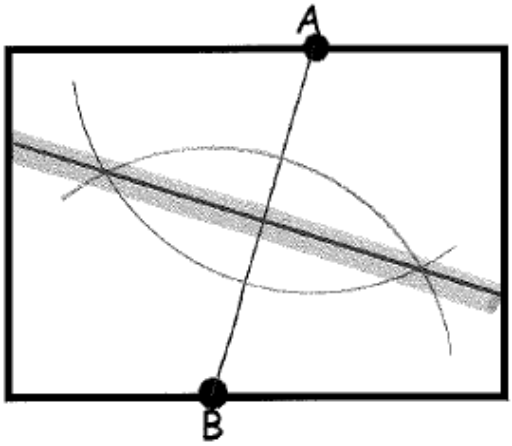
Method 1	*Method 2*
*n*th term of lengths is $n + 3$	*n*th term of 12 20 30
*n*th term of widths is $n + 2$	$= n^2 + 5n + 6$
Area is $(n + 3)(n + 2)$	
$n^2 + 3n + 2n + 6$ $= n^2 + 5n + 6$	

20) *Workings in the table below*

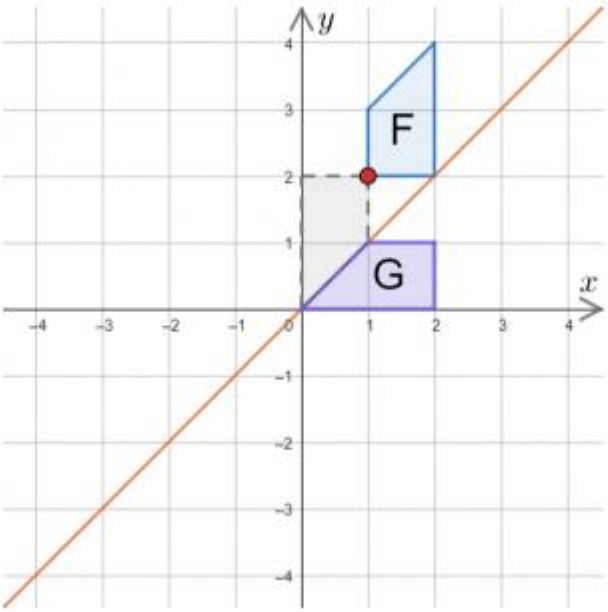
(a)	$a + 9b = 35$ $a + 15b = 59$ $6b = 24$ $b = 4$ $a = -1$
(b)	3 11 19 … … $8n - 5$

Transformations and Loci

21)

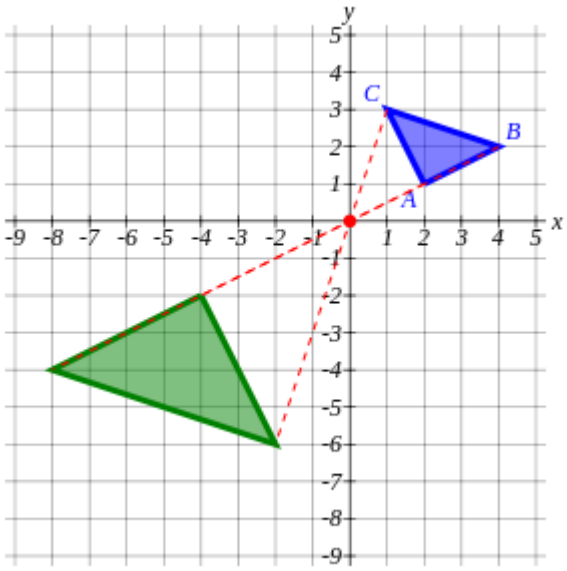


22)



Extension answer – There are no invariant points under the two transformations.

23) *Enlargement scale factor -2 centre (0,0)*



Vectors – Straight line vectors questions

24) a) $\overrightarrow{OP} = 2\boldsymbol{a} + 4\boldsymbol{c}$

b) $\overrightarrow{OM} = 3\boldsymbol{a} + 6\boldsymbol{c}$ $\overrightarrow{OP} = 2\boldsymbol{a} + 4\boldsymbol{c}$

$\overrightarrow{OM} = \frac{3}{2}\overrightarrow{OP}$ *therefore it is a straight line.*